Homework #1
CS170, Spring 2025

Due Monday, Feb 3, at the beginning of class

General Instructions. Please start early, and allow at least 10 hours of work for this homework. Please keep in mind the late day policy on the course website.

Collaboration. Please carefully check the collaboration policy on the course website. When in doubt, ask an instructor.

Consulting outside sources. Please carefully check the policy regarding outside sources on the course website. Again, when in doubt, ask an instructor.

Submission. UPDATED: Homework submission will be via Gradescope — instructions can be found on Piazza. If you choose to type up your homework, we highly recommend using LaTeX (the text markup language we are also using for this assignment). You will likely need to learn LaTex eventually, so might as well start now. That being said, handwritten homework, or homework prepared using any word-processing software, is perfectly fine.

Problem 1. (10 points)
Use the pigeonhole principle to solve the following problems.

(a) [2 points]. Prove that among a group of 311 people, there are at least 11 who are born on the same day of the month (e.g., the 21st or the 12th, etc.). Is the same fact true if there are only 310 people?

(b) [4 points]. Prove that among any $n + 1$ integers you can find two integers so that their difference is divisible by n. You will want the "holes" to be the remainder when you divide a value by n.

(c) [4 points]. You randomly select k distinct integers between 1 and 40, inclusive. What is the smallest k that guarantees that at least one pair of the selected integers will sum to 41? Make sure to be very explicit about what the pigeons and pigeonholes are!

Problem 2. (6 points)
Write the negations of the following statements, in English:

(a). If a number is an integer, it is either odd or even (or both).

(b). All prime numbers other than 2 are odd.

(c). There is a pair of USC students who were born more than an hour apart and less than 3 hours apart.

Problem 3. (6 points)
Express the following in propositional logic, using the operators $\wedge$, $\vee$, and $\neg$.

(a). A statement which is True iff at least two of p, q, and r are False.

(b). A statement which is True iff exactly one of p, q, and r is True.

(c). A statement which is True iff p, q, and r are all True or are all False.

Problem 4. (18 points)

Imagine you are developing a game, and the game designer gives you the following specifications. Translate them into propositional logic using the propositions p = "the player dies", q = "the player opens the door", r = "the player has an upgraded weapon", and s = "the player wins".

(a) [2 point]. If the player opens the door, they either die or win the game (but not both).

(b) [2 point]. The player cannot open the door without having an upgraded weapon.

(c) [2 point]. It is also possible for the player to win the game without opening the door, but only if they do have an upgraded weapon.

Your colleague comes up with a list of statements that he thinks will always be true under the designer's specifications. For each statement: (i) translate it into propositional logic, then (ii) use a counterexample to demonstrate that your colleague is incorrect.

(d) [4 points]. The only way for the player to die is by opening the door.

(e) [4 points]. If the player has an upgraded weapon, they definitely win the game.

(f) [4 points]. It is impossible for the player to both die and win the game.

Problem 5. (9 points)
Re-write the following formulae using only $\wedge$ and $\neg$ (and parentheses if needed).

(a). $p \vee (\neg q) \vee r$

(b). $(p \oplus q) \implies r$

(b). $p \iff (q \vee r)$

Problem 6. (18 points)
Prove the following statements using the rules of inference of propositional logic. You may also use simple logical equivalences such as the laws of commutativity, associativity, and De Morgan's laws. (See the course website for the list of rules of inference).

(a) [4 points].

$$\begin{array}{l} p \implies (q \wedge r) \\ \neg r \\ \hline \therefore \neg p \end{array}$$

(b) [4 points].

$$\begin{array}{l} p \implies q \\ p \implies \neg q \\ \hline \therefore \neg p \end{array}$$

(c) [4 points].

$$\begin{array}{l} p \implies r \\ q \implies r \\ \hline \therefore p \vee q \implies r \end{array}$$

(d) [4 points]. The converse of (c).

(e) [2 points]. What do (c) and (d) allow you to conclude about the relationship between the formulas $(p \implies r) \wedge (q \implies r)$ and $p \vee q \implies r$?

Problem 7. (10 points)
Suppose you start with premises A and B, and prove conclusion C using the rules of inference of propositional logic.

(a). Is $A \wedge B \implies C$ a tautology? Why or why not?

(b). Is $A \wedge B \wedge \neg C$ satisfiable? Why or why not?

(c). Are A, B, and $\neg C$ consistent? Why or why not?

(d). Can you guarantee that A, B, and C are consistent? Why or why not?

(c). Can you be sure that A and B are consistent? Why or why not?

(e). If $C = p \wedge \neg p$, what can you say about the consistency of A and B?